

P425/2
APPLIED
MATHEMATICS
Paper 2
25 July 2018
3 hours



ENTEBBE JOINT EXAMINATION BUREAU

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in Section **A** and **only five** from Section **B**.*

***All** the necessary working **must** be shown clearly.*

Begin each question on a fresh page.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g to be 9.8 ms^{-2} .

SECTION A: 40 Marks

Answer all questions in this Section.

1. Events X and Y are such that $P(X) = \frac{2}{5}$ $P(X \cap Y') = \frac{3}{10}$ and $P(X'/Y) = \frac{3}{5}$

Find: (i) $P(X \cap Y)$

(ii) $P(X/Y)$

(05 marks)

2. A biased coin is tossed 7 times. The ratio of a head to a tail is 1 : 2. Find the:

(i) probability of getting at least 5 heads.

(ii) mean and variance of heads.

(05 marks)

3. Using 2010 as the base year, the price relatives of a commodity in 2011 and 2012 were 115 and 125 respectively. Calculate the price relatives of

(a) 2010

(b) 2012 using 2011 as the base year.

(05 marks)

4. Use the trapezium rule with 6 ordinates to estimate correct to 3 significant figures.

$$\int_0^2 \ln(x^2 + 3) dx$$

(05 marks)

5. The table below shows the estimated radius, r (mm) of an egg laid by a hen of a certain farm depending on the mass, m (g) of the layers mash ratio it is fed on.

Mass m(g)	200	250	330	410	440	500
Radius r (mm)	15.1	17.1	18.1	20.05	20.5	23.1

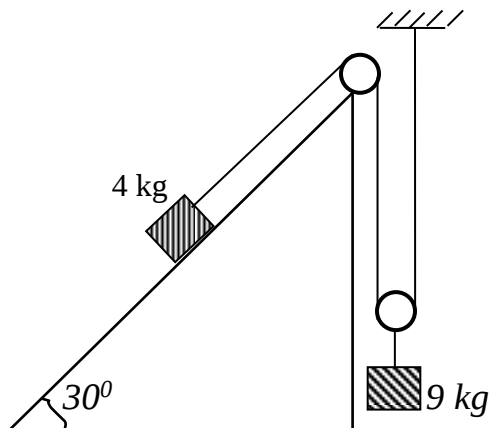
Use linear interpolation or extrapolation to estimate the:

(i) optimal amount of food the hen should be given if it lays an egg of average radius 19.1 mm.

(ii) radius of an egg if the feed ratio supplied is 540 g.

(05 marks)

6. Two masses of 4 kg and 9 kg are connected by a light inextensible string passing over a smooth pulley at the edge of the smooth inclined table and under a smooth movable pulley as shown in the diagram below.



Calculate the

- (i) tension in the string
- (ii) acceleration of the 4 kg and 9 kg masses.

(05 marks)

7. A cyclist travelling at a constant speed of 36 kmhr^{-1} passes a motor car just as it begins to move in the same direction. The car maintains an acceleration of 0.4 ms^{-2} for 40 s and then moves uniformly. Determine the

- (i) distance from the starting point the car will have made before catching up with the cyclist;
- (ii) time taken by the car up to this point.

(05 marks)

8. Four forces acting on the particle are represented by $2\mathbf{i} + 3\mathbf{j}$, $4\mathbf{i} - 7\mathbf{j}$, $-5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{i} - \mathbf{j}$ and act at positions $(1, -5)$, $(-2, -3)$, $(4, 5)$ and $(-1, -3)$ respectively. Determine the

- (i) moments about the origin;
- (ii) hence the line of action.

(05 marks)

SECTION B (60 marks)

Answer any **five** questions from this Section.

9. The cumulative distribution function of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & , \quad x \leq 0 \\ \frac{x^2}{3} & , \quad 0 \leq x \leq a \\ \frac{1}{6}(6x - x^2 - 3), & a \leq x \leq b \\ 1 & , \quad x \geq b \end{cases}$$

- Find (i) the values of a and b
 (ii) $P(0.5 < x < 2.3)$
 (iii) the mean of x

(12 marks)

10. Particles P and Q have the following position and velocity vectors.

Particle	Position vectors	Velocity vectors
P	$\begin{pmatrix} 10 \\ 5 \\ 8 \end{pmatrix} \text{ m}$	$\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \text{ ms}^{-1}$
Q	$\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} \text{ m}$	$\begin{pmatrix} 2 \\ 7 \\ 0 \end{pmatrix} \text{ ms}^{-1}$

Determine

- (i) the displacement of Q relative to P at time t seconds.
 (ii) the time of collision if P and Q collide.
 (iii) the position of collision from the origin of particle P .

(12 marks)

11. Given that $x = 5$, $y = 14$ and $z = 8$ all to the nearest integers, find the maximum value, minimum value, absolute error and percentage error in

(i) $\frac{x+z}{yx}$

(ii) $\frac{xyz}{x-y}$

(iii) $\frac{xz}{y} - \frac{y}{zx}$

(12 marks)

12. 100 students of a certain school sat an examination that was marked out of 135. Below are the marks and numbers of students.

Marks	45-	55-	65-	75-	85-	95-	105-	115-	125 – 135
Number of students	1	1	2	6	21	29	24	12	4

- (a) Calculate the:
- mean and standard deviation.
 - 80th percentile.
- (b) Construct an ogive and from it estimate the
- median mark;
 - number of students with a mark more than 73.
 - third decile.

(12 marks)

13. (a) Derive the simplest iterative formula based on Newton – Raphson method for the equation $3\cos x - x = 0$.
- (b) Construct a flow chart that:
- reads $x_0 = 1.2$, $n = 1$ where x_0 is the first approximation and n is the number of iterations.
 - limits accuracy to less than 5×10^{-4} .
 - prints the root x and number of iterations n .
- (c) Perform dry run for the flow chart in (b) above and correct the root to 3 decimal places.

(12 marks)

14. A particle of mass 2 kg executes simple Harmonic motion about the origin O . When the particle is 1.5 m from O , its speed is $Y \text{ ms}^{-1}$ and the resultant force acting on it is 12 N. Find the
- period and amplitude of motion.
 - least time the particle takes to reach Q .
 - kinetic energy of the particle as it passes through O .

(12 marks)

15. A bag A contains 4 green and 5 red marbles while bag B contains 4 green and 3 red marbles. If the result of tossing a coin is a head then two marbles are picked one at a time without replacement from bag A, otherwise the marbles are selected from bag B. It is known that a head is twice as likely to occur as a tail,
- (a) Find the probability that:
 - (i) both marbles are of the same colour;
 - (ii) the marbles are from bag A given that they are of the same colour.
 - (b)
 - (i) Construct a probability distribution for the number of red marbles drawn.
 - (ii) Find the mean number of red balls.

(12 marks)

16. A particle of weight 49 N is in contact with a horizontal table connected by a light inelastic string passing over a smooth light pulley fixed at the edge of the table. The other end of the string carries another particle B of mass 2 kg hanging freely. The system is released from rest and after 2 seconds, A collided and coalesced with a stationary particle of mass 0.1 kg at rest on the table. If the coefficient of friction between the table and weight is 0.25,

- (a) Calculate the:
 - (i) acceleration of the system,
 - (ii) tension in the string before collision
- (b) Find the change in kinetic energy of A immediately after collision.

(12 marks)